SIMPLIFIED TURBULENT BOUNDARY LAYER COMPUTATION OF SOME DIVERGENT INTERNAL FLOWS

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1. Introduction

Models frequently used for the hydraulic machines are composed of a disc and two perpendicular radial plates in the flow produced either by a linear source (Fig. 1a) or by a linear pit. Or, neglecting, in the first approximation, the interaction between the disc and two plates, in this case two boundary layers can be considered separately [8]: the planar boundary layer between two divergent (or convergent) plates and the axisymmetric boundary layer on the disc (in the presence of a funnel above the disc (Fig. 1b) to vary slowing down or acceleration of the flow).

To calculate the flows of this type (Figures 1a and 1b), we are going to develop herein a simplified method by using a new variant of the phenomenologic semi-empiric turbulent boundary-layer theory (Novozilov [1]) that is founded on the analogy with the rheologic power-laws largely utilized in the study of non-Newtonian non-linear flows and on the use of the Karman turbulence model. It is to note also that the viscous sublayer is neglected as well as the layer situated over the zone for which the universal logarithmic law is usually used.

2. Planar (or axisymmetric) turbulent boundary layer

The equations of the planar turbulent boundary layer can be written in the following form:

$$\left.\begin{aligned} \bar{u}\frac{\partial \bar{u}}{\partial x} + \bar{v}\frac{\partial \bar{u}}{\partial y} &= u_e u_e' + \frac{1}{\rho}\frac{\partial \tau}{\partial y} \\ \frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} &= 0 \end{aligned}\right\} \qquad (1)$$

where (x, y) – usual orthogonal curved coordinates, $(\bar{u}, \bar{v})$ –velocity components at (x, y) in the boundary layer, $u_e(x)$ – external velocity out of the frontier of the boundary layer, $\tau_t = \mu\frac{\partial \bar{u}}{\partial y} - \rho\overline{u'v'} = \mu\frac{\partial \bar{u}}{\partial y} + \tau$ – global turbulent stress where the viscous part is neglected.

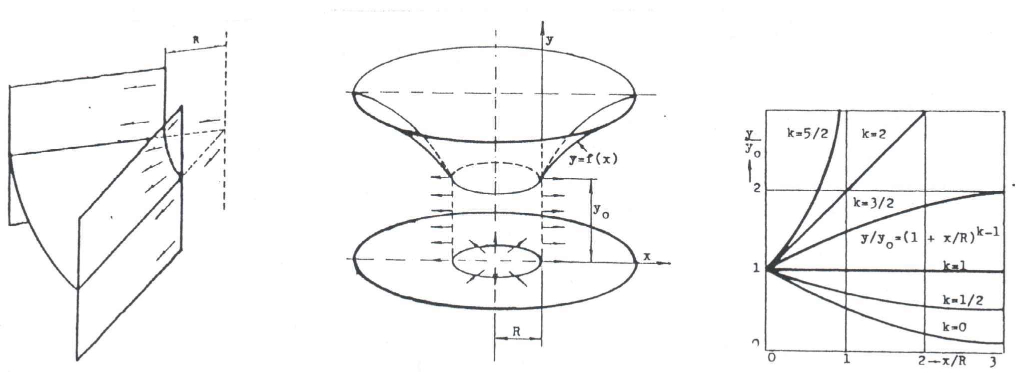

Fig. 1. Model of divergent rectangular channel (a), axisymmetrical diffusor (b) and meridian line shapes (c).

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Abstract

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Due to the rheological power law and the Prandtl turbulence model chosen herein, it results that:

$$\frac{1}{\rho}\tau = \nu k_n T^n \frac{\partial \bar{u}}{\partial y} , \qquad (2)$$

$$T = \frac{y^2}{\nu}\left|\frac{\partial \bar{u}}{\partial y}\right|. \qquad (3)$$

The system of equations (1) must be resolved for the following boundary conditions:

$$\left.\begin{aligned} &\bar{u} = \bar{v} = 0, \quad \frac{\partial \bar{u}}{\partial y} \to \infty, \quad \text{for } y = 0 \\ &\bar{u} = u_e(x), \quad \tau = 0, \quad \text{for } y = \delta(x) \end{aligned}\right\} \qquad (4)$$

as well as for the next initial condition:

$$\bar{u} = u_0(y) \quad \text{for} \quad x = x_0 , \qquad (5)$$

where $\delta(x)$ is the thickness of the boundary layer.

In order to calculate the turbulent boundary layer, we used in practice most often an integral relation obtained from the system of equations (1), called the momentum equation, in the following form:

$$\frac{d\delta_2}{dx} + \frac{u'_e\,\delta_2}{u_e}(2+H) = \frac{1}{2}c_f \qquad (6)$$

where

$$\delta_1(x) = \int_0^\delta (1-\frac{\bar{u}}{u_e})dy, \quad \delta_2(x) = \int_0^\delta \frac{\bar{u}}{u_e}(1-\frac{\bar{u}}{u_e})dy, \quad c_f = \frac{2\tau_0}{\rho u_e^2}, \quad H = \frac{\delta_1}{\delta_2}. \qquad (7)$$

By analogy with the Novozilov's procedure [1], the practical approximate method of computing the turbulent boundary layer proposed herein consists in the following steps to be done:

– At first, for a given distribution of the external velocity $u_e(x)$, a Riccati like differential equation:

$$\frac{dz}{dx} - 23{,}163\frac{\nu}{u_e^3}(u'_e)^2 z^2 + 2{,}097\frac{u'_e}{u_e}z = 0{,}063\frac{u_e}{\nu} \qquad (8)$$

is to be integrated with a prescribed initial condition of δ_2 for $x = x_1$:

$$z(x) = (\frac{u_e\delta_2}{\nu})^{4/3} = z_1 \quad \text{for} \quad x = x_1. \qquad (9)$$

– After finding z(x), determine the momentum thickness by using (10):

$$\delta_2(x) = \frac{\nu}{u_e}z^{3/4}, \qquad (10)$$

as well as the parameter Q(x), given by:

$$Q(x) = \frac{\nu}{u_e^2}u'_e z(x). \qquad (11)$$

– The next step is to determine the functions E(Q), G(Q) and H(Q), defined as follows:

$$E(Q) = (2-n)\left\{\left[1+H(Q)\right]Q - \frac{1}{2}G(Q)\right\} + 2Q, \tag{12}$$

$$G(Q) = c_f\left(\frac{u_e\delta_2}{\nu}\right)^{1-n}, \tag{13}$$

$$H(Q) = \frac{\delta_1}{\delta_2}. \tag{14}$$

It is to be noticed that the function E(Q) depends on n, not only explicitely, but also through functions G(Q) and H(Q), the forms of which depend on n and can be determined numerically. In this sense, it is possible to ascertain that for every particular value of n, there is "its proper formula" (12). In the particular case (n = 2/3, $k_n = 0,55$) and for the Prandtl turbulence model (3), we carried out detailed computations of the functions E(Q), G(Q) and H(Q) with approximate analytical expressions of the next forms:

$$\left.\begin{aligned} E(Q) &= -0.063 + 4.097Q - 23.163Q^2 \\ G(Q) &= 0.0938 + 2.1143Q + 36.1035Q^2 + 984.7348Q^3 \\ H(Q) &= 1.6529 - 2.17Q \end{aligned}\right\} \tag{15}$$

– So, the formulas:

$$H(x) = H(Q), \quad c_f(x) = z^{-1/4}G(Q) \tag{16}$$

offer two particularly important characteristics of the turbulent boundary layer.

– Finally, by using the well known simplest idea, it will be possible to calculate an improved approximation of the mean velocity in the turbulent boundary layer by (17):

$$\frac{\bar{u}}{u_e} = \left[\frac{y}{\delta_2}\frac{H-1}{H(H+1)}\right]^{(1/2)(H-1)}, \tag{17}$$

where the functions $\delta_2(x)$ and H(x) are determined by the formulas (10) and (16).

Applying the analogous procedure as above in the planar case, we can obtain also the practical approximate method for the axially symmetric turbulent boundary layer. The only difference appears in Eq. 8 where a new term $\frac{4}{3}\frac{r'}{r}z$ should be added at left side (r is the radius of the sections taken at right angles to the axes of revolution).

3. Elementary case

3.1. Planar turbulent boundary layer between two divergent surfaces. It is the question of two divergent surfaces (i.e. two radial plates for k = 1, Fig. 1a) in the flow produced by a linear source, so that the external velocity can be written in the form:

$$u_e(x) = U_0\left(1+\frac{x}{L}\right)^{-k}, \quad x \ge 0, \quad k \ge 0. \tag{18}$$

If we replace (18) into (8), then it results:

$$\frac{dz}{dx} = 23,163\frac{k^2}{LR_e}\left(1+\frac{x}{L}\right)^{k-2}z^2 + 2,097\frac{k}{L}\left(1+\frac{x}{L}\right)^{-1}z + 0,063\frac{R_e}{L}\left(1+\frac{x}{L}\right)^{-k}, \tag{19}$$

where $R_e = U_0 L/\nu$. First, we found the analytical solution of the Riccati differential equation (19), for every other values of k, except for:

$$0{,}18139<k<1{,}46844, \tag{20}$$

in the form:

$$z(x) = \frac{LR_e}{23{,}163k^2}(1+\frac{x}{L})^{2-k} M(x) + N(x), \tag{21}$$

$$M(x) = \frac{1}{2}\frac{1}{L+x}\frac{(w-1)-Cw(w+1)(L+x)^w}{1+Cw(L+x)^w}, \quad N(x) = \frac{1-1{,}5485k}{23{,}163k^2} R_e (1+\frac{x}{L})^{1-k},$$

where the integration constant C, in accordance with the initial condition z(0) = 0, is the following:

$$C = \frac{w-3{,}097k+1}{w(w+3{,}097k-1)} L^{-w}, \qquad \text{with} \qquad w = (1-6{,}194k+3{,}75432k^2)^{1/2}.$$

Then, for the values of k, situated into the interval (20), we found the solution in the form:

$$Q(x) = 0{,}06685 - \frac{0{,}02158}{k} - p\,\mathrm{tg}\left[23{,}163kp\ln(\frac{x+L}{C_0})\right], \tag{22}$$

$$p = (-0{,}00175 + \frac{0{,}00288}{k} - \frac{0{,}00046}{k^2})^{1/2}, \quad C_0 = L\exp(-\frac{1}{23{,}163kp}\mathrm{arctg}\frac{0{,}06685k-0{,}02158}{kp}).$$

3.2. Turbulent boundary layer in an axisymmetric diffusor. In reality, it is the question of the flow produced by a linear source on a disc in the presence of a perpendicularly placed funnel (Fig. 1b). The equation of the meridian line of the funnel is:

$$y = y_0(1+\frac{x}{R})^{k-1}, \tag{23}$$

where $y_0 = y(0)$, $x \geq 0$, while the distributions of $u_e(x)$ and r(x), for different values of the exponent k (Fig. 1c), are:

$$u_e(x) = U_0(1+\frac{x}{R})^{-k}, \; r(x)=R+x, \tag{24}$$

where $U_0 = u_e(0)$ and $x \geq 0$.

Repeating the analogous procedure as in §3.1, we determined the both functions z(x) and Q(x).

4. Calculation of the global turbulent boundary layer characteristics

After determining the functions z(x) and Q(x), we can calculate now all global boundary-layer characteristics in accordance with the order of operations announced at §2. So, in the case of the planar diffusors, we are choosing herein k = 1, i.e. the case of two radial divergent plates (Fig. 1a). Or, while this value k = 1 is situated over the interval (20), then the solution is done by (22), with p=0,0258 and C_0=0,17L. Next, the relation (11) offers z(x), then the momentum thickness $\delta_2(x)$ is obtained from (10). Afterwards, the calculation goes on in accordance with the order of operations above-mentioned. The turbulent boundary layer characteristics, calculated in this way in the particular case $R_e = 5.10^5$, are presented in Figs. 2 and 3. It will be particularly remarked that, the turbulent boundary layer separation (G(Q)=0) occurs nearly at x = 0,465L. It is to be noticed that in the analogous laminar case [3], the separation point appeared at x = 0,161L.

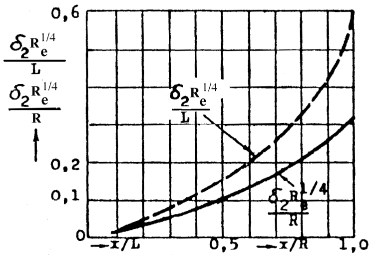

Fig. 2. Variation of adimensional momentum thickness.

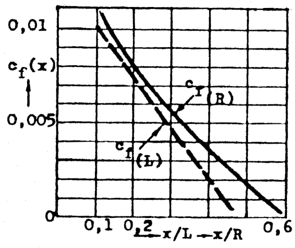

Fig. 3. Variation of skin-friction adimensional coefficient.

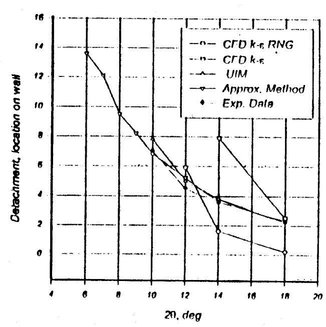

Fig. 4. Comparison of the detachment point.

Concerning the axisymmetric diffusors, we are choosing herein k=1, i.e., after (23) and (24), the case of an annular disc in the flow produced by a linear source (Fig. 1b). The turbulent boundary layer characteristics, calculated in a similar way as above, in the particular case $R_e = 5.10^5$, are given by Figs. 2 and 3. The turbulent boundary layer separation (G(Q) = 0) appears at x = 0,6R, while in the analogous laminar case [3], the point of boundary layer separation occurred at x = 0,178R.

5. Comparison with the other methods and models of turbulence

Figure 4 shows our results [4] concerning the position of detachment points, together with the corresponding results obtained by two models of turbulence: $\mathrm{CFD} - k - \varepsilon - \mathrm{RNG}$, $\mathrm{CFD} - k - \varepsilon$ by Rothe et al. [5], and UIM of Stanford University by Johnston [6] as well as the experimental results of Ashjaee et al. [7] It can be remarked that the approximate method predicts the detachment point with the accuracy of 6 % approximately for the angles 10° to 18°.

6. Remarks on some (divergent) external flows

The simplified practical method is applicable for both internal and external flow problems. So, an approximate procedure is proposed to evaluate the evolution of the velocity profile in a neighborhood of the separation point of the turbulent boundary layer in the case of the Prandtl turbulence model [8]. The Fig. 5 gives the calculated velocity profile for one section after the separation point compared to the experimental results (presented by the black points) of Mises in the case of the ident 3800 [2], while the Fig. 6 shows the calculated velocity profile by the developed simplified method quite nearly the wall in the same case.

7. Conclusion

By beginning with the integral momentum equation and with the analogy with the rheological power law, which is effectivelly used to study flow of non-Newtonian liquids with nonlinear viscosities, a simple practical method is obtained for an approximate computation of both turbulent boundary layers, planar and axisymmetric, in the case of the Prandtl model of turbulence, where for the process ending it is necessary to know only two empirical constants (for example, n = 2/3 and $k_n = 0,55$). The developed method is applicable to some internal flows, such as a divergent rectangular channel (Fig. 1a) or an axisymmetrical diffusor (Fig. 1b),

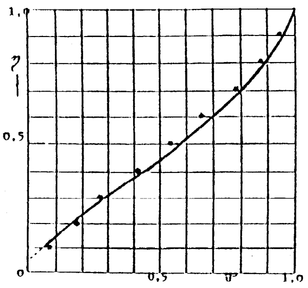

Fig. 5. Calculated velocity profile compared to the experimental results.

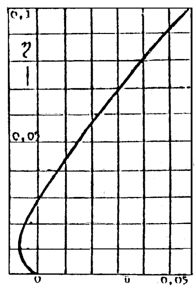

Fig. 6. Calculated velocity profile nearly to the wall.

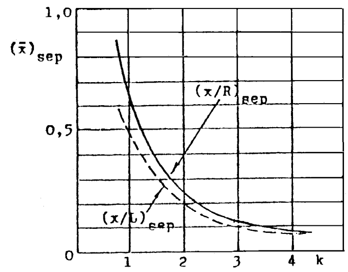

Fig. 7. Turbulent boundary layer separation on the side wall and on the bottom of the channel.

as well as other planar or axisymmetrical, rectilinear or curvilinear, diffusors [4]. So, the Fig. 7 summarizes some obtained results [8] in the case of the divergent rectangular channel for $R_e = 5.10^5$. From there, the turbulent boundary layer separation appears earlier on the side wall (dotted line) than on the bottom of the channel, independently of the slowing down of the flow, which is in accordance with the same tendency, well-known before [3], in the analogous laminar case.

The method gives relatively good results for all global characteristics δ_2, H and c_f of the turbulent boundary layer in both, internal and external, problems. As expected, the velocity profiles are of the lower quality, relatively schematized. In return for this drawback, the method enables a satisfactory evolution evaluation of the flow velocity profile just upstream and downstream the separation point of the turbulent boundary layer (Figs. 5 and 6). Finally, it can be noted that we did also some comparaisons between the actual practical simplified method and the corresponding experimental results [7] with satisfactory concordance.

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